
Q.1 (7 points) Affamés

Give your answers in Spanish, German, Italian, or French using a minimum of 30 words.



En la comida de la Asamblea Internacional de Matemáticas sin Fronteras en Alsacia, un brasileño, un suizo y un alemán cenan juntos y con sus esposas.

Cuando hacen el pedido, un hombre y una mujer piden cada uno una tarta flambeada, un hombre y una mujer piden cada uno una ensalada alsaciana y un hombre y una mujer piden cada uno caracoles.

La esposa del alemán ha elegido una tarta flambeada. El brasileño, que ha elegido una ensalada alsaciana, dice entonces: «¡Qué curioso, en cada pareja, el hombre y la mujer han elegido platos diferentes!».

Con la ayuda de estos datos, indica lo que ha elegido cada hombre. Justifica la respuesta.

Durante la cena dell'Assemblea Internazionale di Matematica senza Frontiere in Alsazia, un brasiliano, uno svizzero e un tedesco cenano assieme con anche le loro mogli.

Quando effettuano l'ordinazione iniziale un uomo e una donna prendono entrambi una *tarte flambé*, un uomo e una donna ordinano entrambi un'insalata alsaziana e un uomo e una donna ordinano entrambi delle lumache.

La moglie del tedesco ha ordinato una *tarte flambé*. Il brasiliano che ha ordinato l'insalata le dice "E' curioso, in ogni coppia, l'uomo e la donna hanno ordinato piatti diversi!"

In base alle informazioni fornite, individuate la scelta di ogni uomo, giustificando la vostra risposta.

Beim Galadinner der internationalen Versammlung von Mathematik ohne Grenzen sitzen ein Deutscher, ein Brasilianer und ein Schweizer zusammen mit ihren Ehefrauen an einem Tisch.

Jeweils ein Mann und eine Frau bestellen einen Flammkuchen, jeweils ein Mann und eine Frau nehmen einen elsässischen Salat und jeweils ein Mann und eine Frau bestellen Schnecken.

Die deutsche Dame hat einen Flammkuchen bestellt, der brasilianische Herr einen elsässischen Salat. Er bemerkt: "Das ist lustig! Bei jedem Ehepaar an diesem Tisch haben Mann und Frau unterschiedliche Gerichte gewählt."

Was hat jeder Mann bestellt? Begründet eure Antwort.

Au repas de l'Assemblée Internationale de Mathématiques sans Frontières en Alsace, un Brésilien, un Suisse et un Allemand d'inent ensemble et avec leurs épouses.

Lorsqu'ils passent commande, un homme et une femme prennent chacun une tart flambée, un homme et une femme prennent chacun une salade alsacienne et un homme et une femme prennent chacun des escargots.

L'épouse de l'Allemand a pris une tarte flambée. Le Brésilien, qui a pris une salade alsacienne, lui dit alors: "C'est curieux, dans chaque couple, l'homme et la femme ont pris des plats différents!"

A l'aide des renseignements donnés, indiquer ce que chaque homme a choisi. Justifiez votre réponse.



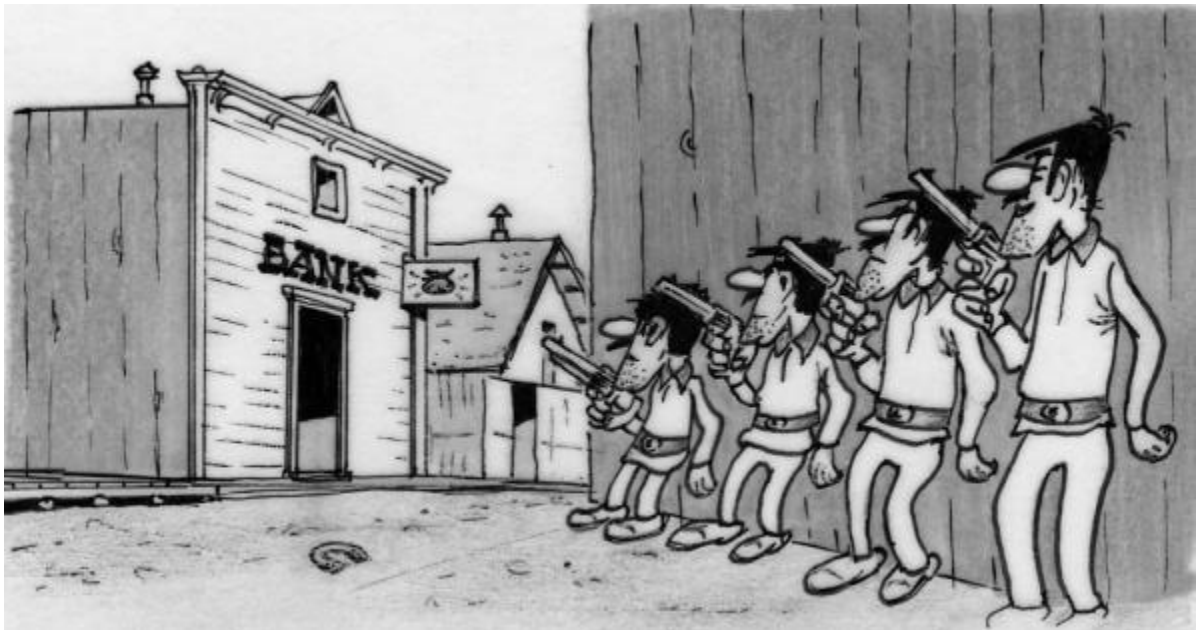
Q.2 (5 points) *The Daltons are Back.*

The Daltons want to rob the bank. Under threat, the cashier gives them clues to help them obtain the safe combination:

- the combination is a sequence of three distinct single digit numbers, arranged in ascending order;
- the sum of the three numbers is 18;
- a product of two of these three numbers added to the third is a square.

What are the possibilities for the combination of the safe?

Justify your answer.



Q.3 (7 points) What Happiness!

Here is a three-step algorithm:

- Step 1: Choose a non-zero positive integer starting number.
- Step 2: Calculate the sum of the squares of its digits and write the result obtained.
- Step 3: Repeat step 2 with the previous result.



By running this algorithm, we get a sequence of numbers.

- If one of the numbers in this sequence is 1, then the algorithm stops and the starting number is a "happy number".
- If one of the numbers in this sequence repeats, then the algorithm stops and the starting number is a "happy number".

Here are two examples:

- by choosing 70 as the starting number:
 $7^2 + 0^2 = 49$; $4^2 + 9^2 = 97$; $9^2 + 7^2 = 130$; $1^2 + 3^2 + 0^2 = 10$ and $1^2 + 0^2 = 1$.

We get 1, so 70 is a "happy number".

- by choosing 40 as the starting number, we obtain the following sequence of numbers:

16 - 37 - 58 - 89 - 145 - 42 - 20 - 4 - 16.

The number 16 is repeated so 40 is a "happy number".

Find the five "happy numbers" less than 20.

Thomas claims that the year 2021 is a "happy year" because 2021 is a "happy number".

Is he correct? If not, what will the next "happy year" be?

Justify your answer.

Q.4 (5 points) All White

Lili plays with chips each having a gray side and a white side and bearing the same number on both sides.



When the game starts, the tokens are arranged as shown in the figure.

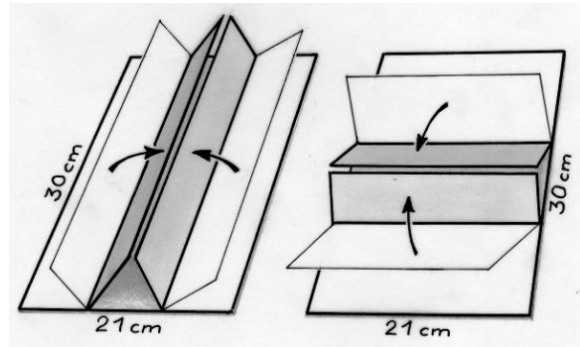
When Lili turns over a token, the two neighbouring tokens also turn over.

The game ends when all of the tokens are white.

How can Lili go about finishing the game with as few flips as possible?

Q.5 (7 points) Many-fold

We want to make two prisms whose base is an equilateral triangle. We have two cardboard sheets of length 30 cm and width 21 cm. The first is folded into three equal rectangles lengthwise and the second into three equal rectangles, but widthwise (see drawing).



Each of the prisms is closed by connecting the two rectangular side faces with adhesive tape.

Calculate the volume of the prism in each of the two configurations.

In which configuration is the volume the greatest?

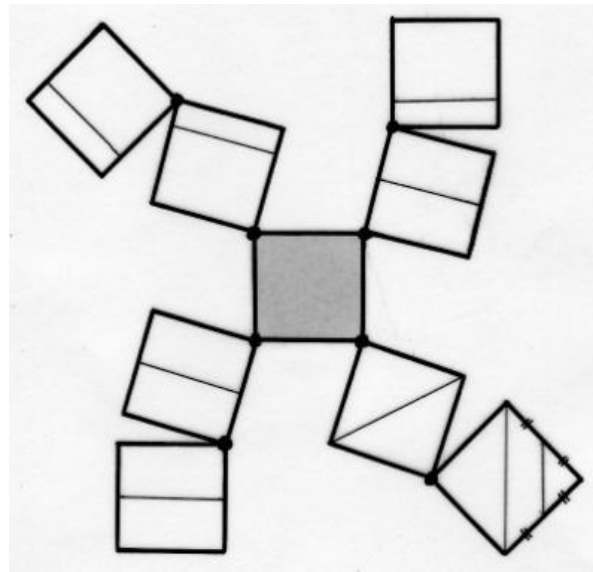
Calculate the ratio of the two volumes.

Q.6 (5 points) Collection Notice

The squares in this figure are drawn on tracing paper. They revolve around one of their vertices so as to overlap and thus exactly cover the central square.

In six of the nine squares, the added segments are parallel to one side and go through the quarter-point or mid-point of the other side.

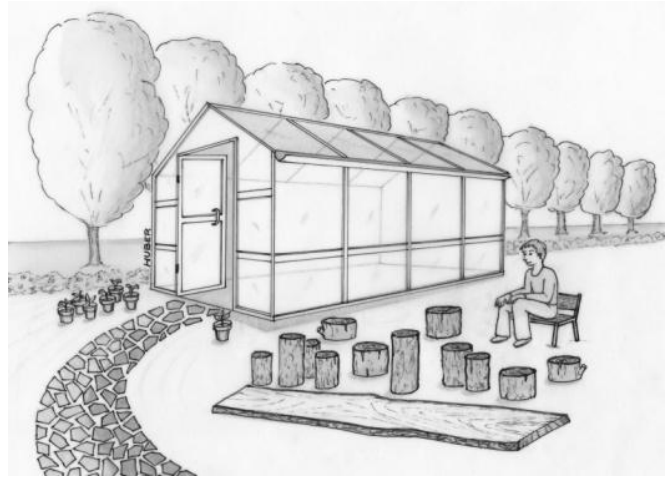
Build the central square with side 8cm which is covered by the eight squares.



In how many regions is the obtained square then divided?

Q.7 (7 points) Log Legs

Éloi has recovered a very long plank from the back of the garden. He thinks it would be perfect for a shelf in his greenhouse. It should be placed on legs so that it is at the right height to place the pots of cuttings in the spring. Éloi finds several logs of various lengths which can be used to make the legs by stacking them.

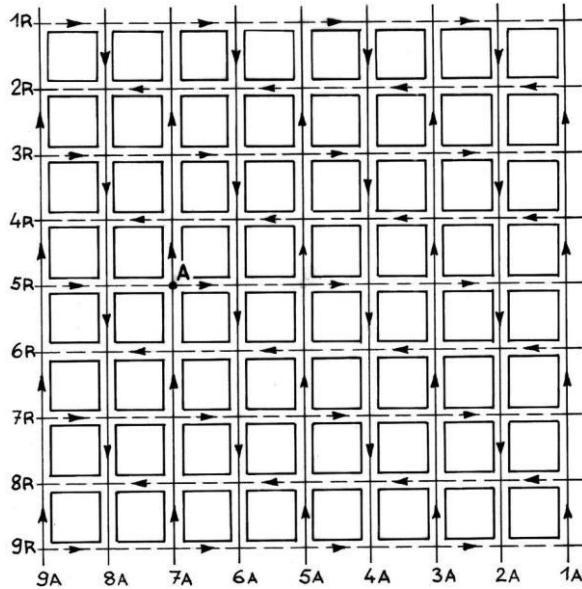


He has two 10cm logs, three of 20cm, four of 30cm, one of 40cm, one of 50cm and one of 60cm. The logs will all be used without cutting, all legs must be the same height, and each leg is made up of at least two logs.

Indicate the number of legs of the shelf and the logs that make up each leg.

Justify your answer.

Q.8 (5 points) Mickey Moves



A large American city is divided into 100 metre-square blocks by numbered Avenues from East to West and by numbered Streets from North to South.

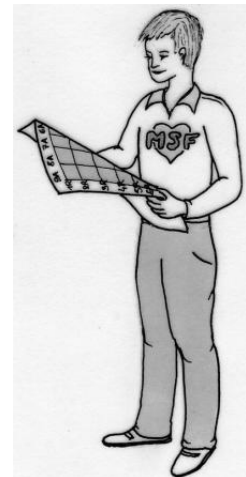
All Avenues and Streets are one-way. The directions of traffic change from one avenue to another and from one street to another; they are shown on the map.

Mickey lives at Junction **A** (**7A; 5R**), at the intersection of 7th Avenue and 5th Street.

He moves home to another junction and makes back and forth trips by car from **A** by the shortest paths. He respects, of course, the directions of movement!

He notes that the "return" distance is double the "outward" distance.

Give the coordinates in the form (xA; yR) of three possible intersections which locate Mickey's new home.



Q.9 Malo Sorted (7 points)

Malo is the President of a big basketball club. To celebrate the club's 50th anniversary, he decides to organize a lottery whose winning ticket will win a trip to Berlin.

To do this, he prints 1,000 tickets numbered from 1 to 1,000.

Looking closely at the tickets, he says:

"There is a small problem with tickets 908 and 806 because two different people could show up with the winning ticket! "



Explain the problem encountered with these two tickets.

Identify all the problematic pairs of tickets.

Q.10 Fractalus Cactus (10 points)

Here is a strange geometric cactus whose trunk is a square of side 5 cm.

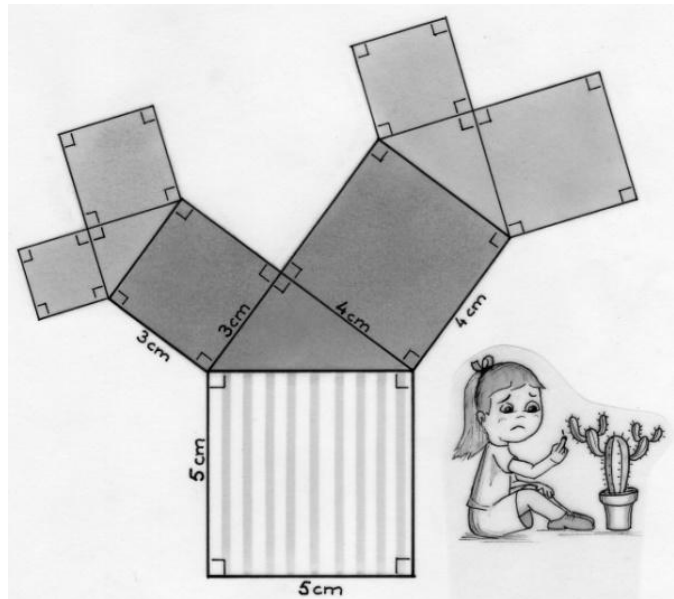
In the first week, the cactus expands with a right-angled triangle with sides 3cm, 4cm and 5cm and two squares, as shown in the drawing. The cactus now has two branches.

In the second week the same process continues: on each branch grows a right-angled triangle, similar to the previous one, and its two associated squares. In the third week, the growth of the cactus continues in the same way.

The cactus is made up of similar squares and right-angled triangles.

Make an accurate drawing of the cactus in the third week.

Calculate the area of each of the squares and colour the squares of the same area with the same colour.



Senior Classes Only

Q.11 It Sticks! (5 points)

Charlotte is going to buy stamps.

"I would like €10 worth of stamps.

I need stamps at €0.10, €0.20 and €0.50." she says.

"Yes, but how many of each?" the employee asks her.

"I need ten times more 10-cent stamps than 20-cent stamps." she replies.



How many stamps of each kind does the employee give her?

Explain your answer.

Q.12 By Boat (7 points)

To circumnavigate a small Caribbean island, Amalio takes an hour in his rowing boat. Tissia, his friend, with her new speedboat, takes 10 min.

The two friends leave from the same place and follow the same route.

When Amalio has made a complete lap, how many laps will Tissia have done?

Justify your answer.

How long till Tissia overtakes Amalio again?

Justify your answer.



Q.13 The Feet Under the Table (10 points)

Laura has assembled the coffee table pictured below. The base was assembled from six identical wooden battens, each perpendicular to two others. Each cleat has the shape of a rectangular parallelepiped 55 cm long, 5 cm wide, and 5 cm thick. The top of this table is a disc. This is attached to the base at the location of points A, B and C which are equidistant from the center of the tabletop. The distance between point A and the edge of the tabletop is 10 cm.

Determine the nature and dimensions of triangle ABC, then calculate the radius of the tabletop.

Justify your answer.

